Knowledge Discovery & Data Mining

– Data Preprocessing –

Data compression & Sampling

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Based on the original version by Professor Jiliang Tang

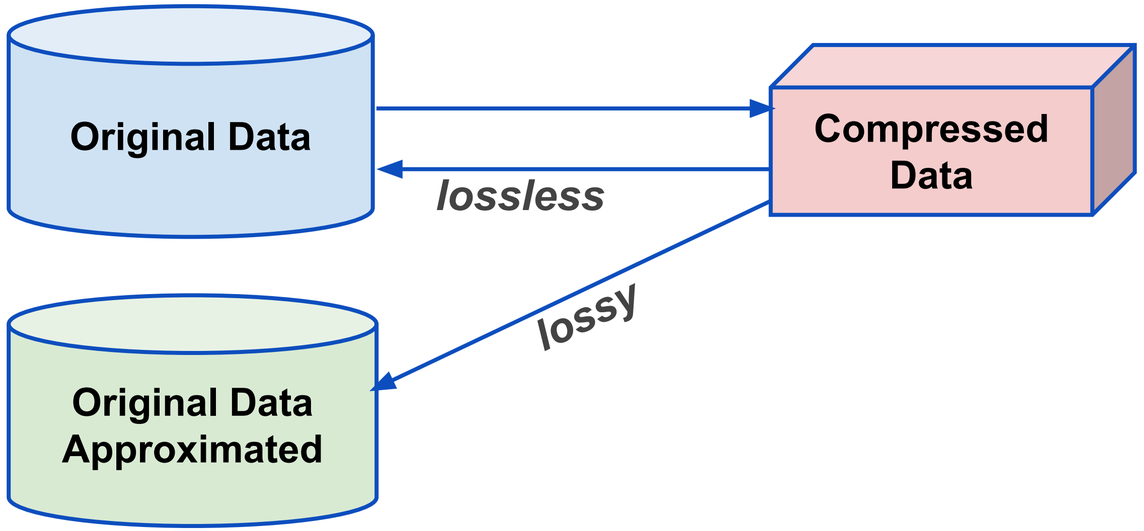
Data Reduction

- Data compression
  - Discrete wavelet transform (DWT)
- Sampling
  - Sampling without replacement
  - Sampling with replacement
  - Cluster or Stratified Sampling

Data Compression

In **data compression**, transformations are applied so as to obtain a reduced or "compressed" representation of the original data.

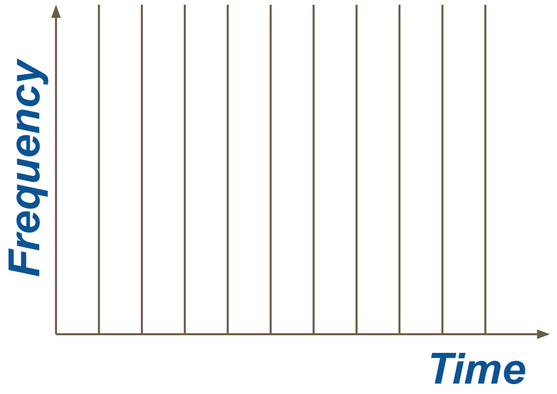
- If the original data can be reconstructed from the compressed data without any information loss, the data reduction is called **lossless**.
- If,we can reconstruct only an approximation of the original data, then the data reduction is called **lossy**



Discrete wavelet transform (DWT)

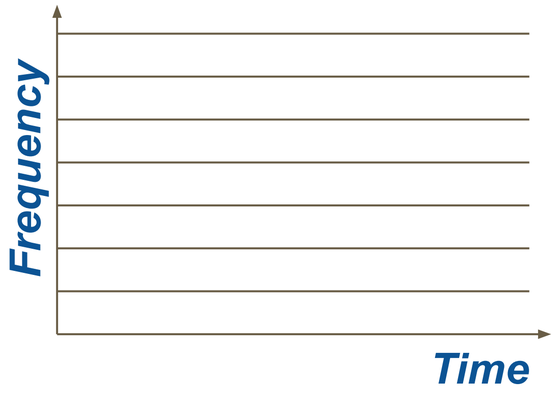
The **discrete wavelet transform (DWT)** is a linear signal processing technique that, when applied to a data vector x, transforms it to a numerically different vector, x’ , of wavelet coefficients.

Time series



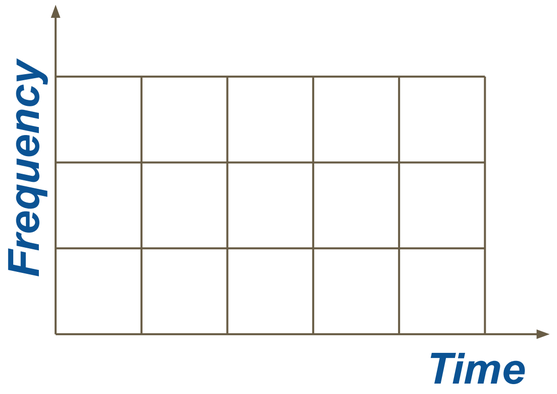
focuses on the raw temporal evolution of data

Fourier transform



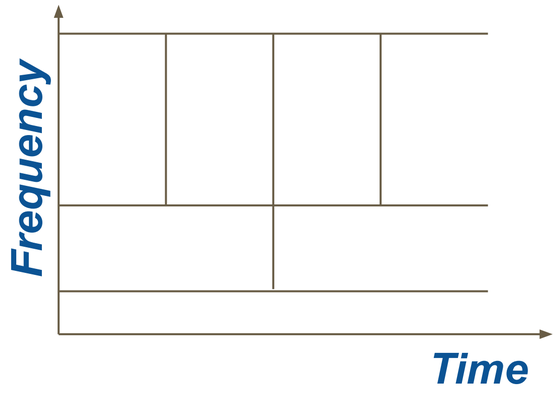
breaks down a signal into its constituent frequencies, but lose the exact timing of these frequencies.

Spectrogram



provides a time-frequency representation, showing when and with what intensity each frequency is present.

Wavelet transform

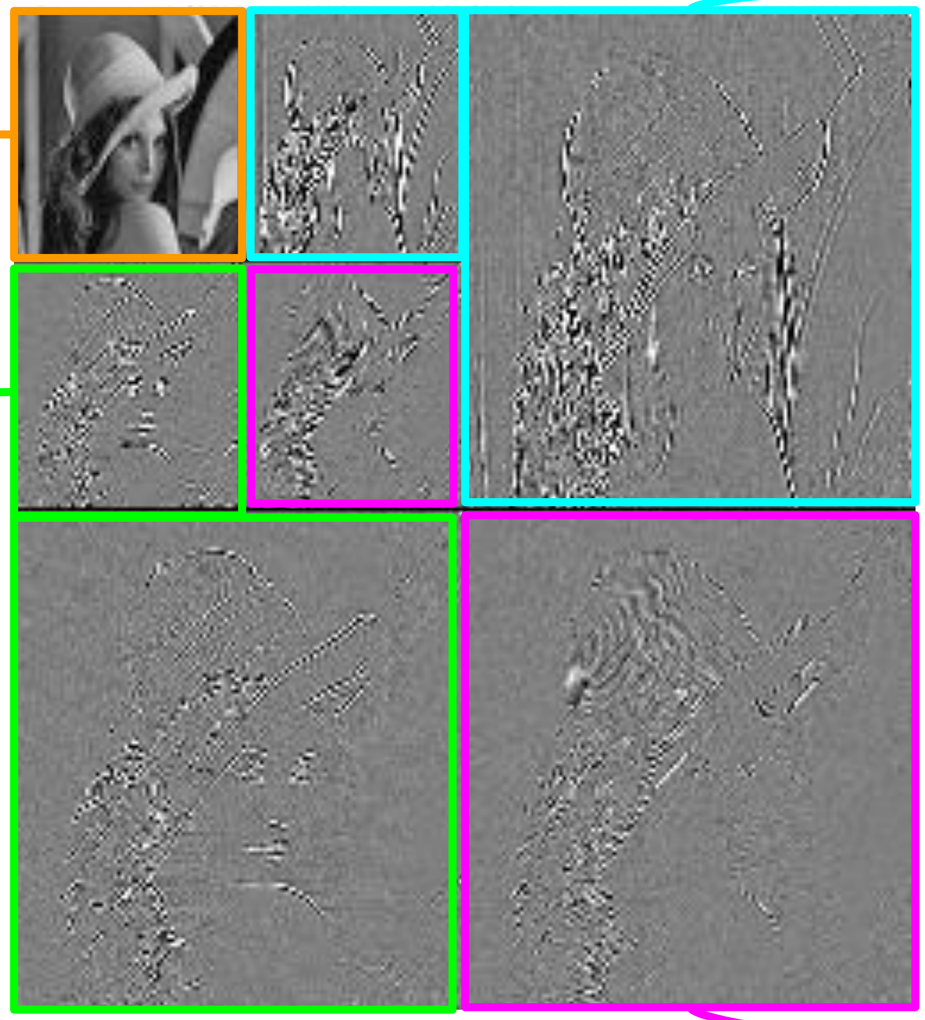


offers a multi-resolution perspective, allowing for variable time and frequency resolution, ideal for signals that have non-stationary or evolving characteristics.

Discrete wavelet transform (DWT)

Used for image compression

A low-resolution of the original image.



Horizontal details of the image. It captures the difference between rows in the image.

Vertical details of the image. It captures the difference between columns in the image.

Diagonal details of the image. It captures the diagonal changes in the image.

Discrete wavelet transform (DWT)

- **Data Preparation**: Ensure the data length L is a power of 2. If not, pad the data with zeros to make the length a power of 2.
- **Transform Functions:** During the transformation, two operations are applied to consecutive pairs of data points:
  - Smoothing (Averaging): Calculate the average of two consecutive points, x_i and x_{i+1}: average = $(x_i + x_{i+1}) / 2$
  - Differencing: Calculate the difference between the same pair of points: difference = $(x_i - x_{i+1}) / 2$
- **Transform Application:** For each pair of consecutive data points, the transformation produces one average value and one difference value. The result is a dataset where:
  - The first half contains the average values (smoothed data),
  - The second half contains the difference values (details), but the overall data length remains the same as the original.
- **Recursive Application for Multi-resolution Analysis:** To analyze the data at different levels of detail, the transform is recursively applied to the average values from the previous step. This process can be repeated multiple times until the dataset is reduced to a single average value.

Discrete wavelet transform (DWT)

Example. Suppose we have a time series S = [2, 2, 0, 2, 3, 5, 4, 4], using DWT, then it can be transformed to S'=

- **Data Preparation**:
  - Ensure data length L is a power of 2. Pad with zeros if necessary..
- **Transform Functions:** For each pair x_i and x_{i+1}:
  - Average (smoothing): avg = $(x_i+x_{i+1}) / 2$
  - Difference (differencing): diff = $(x_i-x_{i+1}) / 2$
- **Transform Application:** (Total length remains unchanged).
  - First half of the result = average values
  - Second half = difference values
- **Recursive Application for Multi-resolution Analysis:**
  - Repeat the transform on the average values for multi-level resolution until the dataset is reduced to a single average value.

Discrete wavelet transform (DWT)

Example. Suppose we have a time series S = [2, 2, 0, 2, 3, 5, 4, 4], using DWT, then it can be transformed to S'=

Resolution	Averages	Detail Coefficients(difference)
8	[2, 2, 0, 2, 3, 5, 4, 4]	
4	[2 , 1 , 4 , 4]	[0 , -1 , -1 , 0]

Suppose we aim to represent S using only L/2 numbers; which four values could best approximate S?

to reconstruct the original list of values

Discrete wavelet transform (DWT)

Example. Suppose we have a time series S = [2, 2, 0, 2, 3, 5, 4, 4], using DWT, then it can be transformed to S'=[**2.75**, **-1.25**, **0.5**, **0**, **0**, **-1**, **-1**, **0**]

Resolution	Averages	Detail Coefficients(difference)
8	[2, 2, 0, 2, 3, 5, 4, 4]	
4	[**2** , **1** , **4** , **4**]	[**0** , **-1** , **-1** , **0**]
2	[**1.5** , **4**]	[**0.5** , **0**]
1	[**2.75**]	[**-1.25**]

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lossless or lossy?

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Where is the Data Compression?

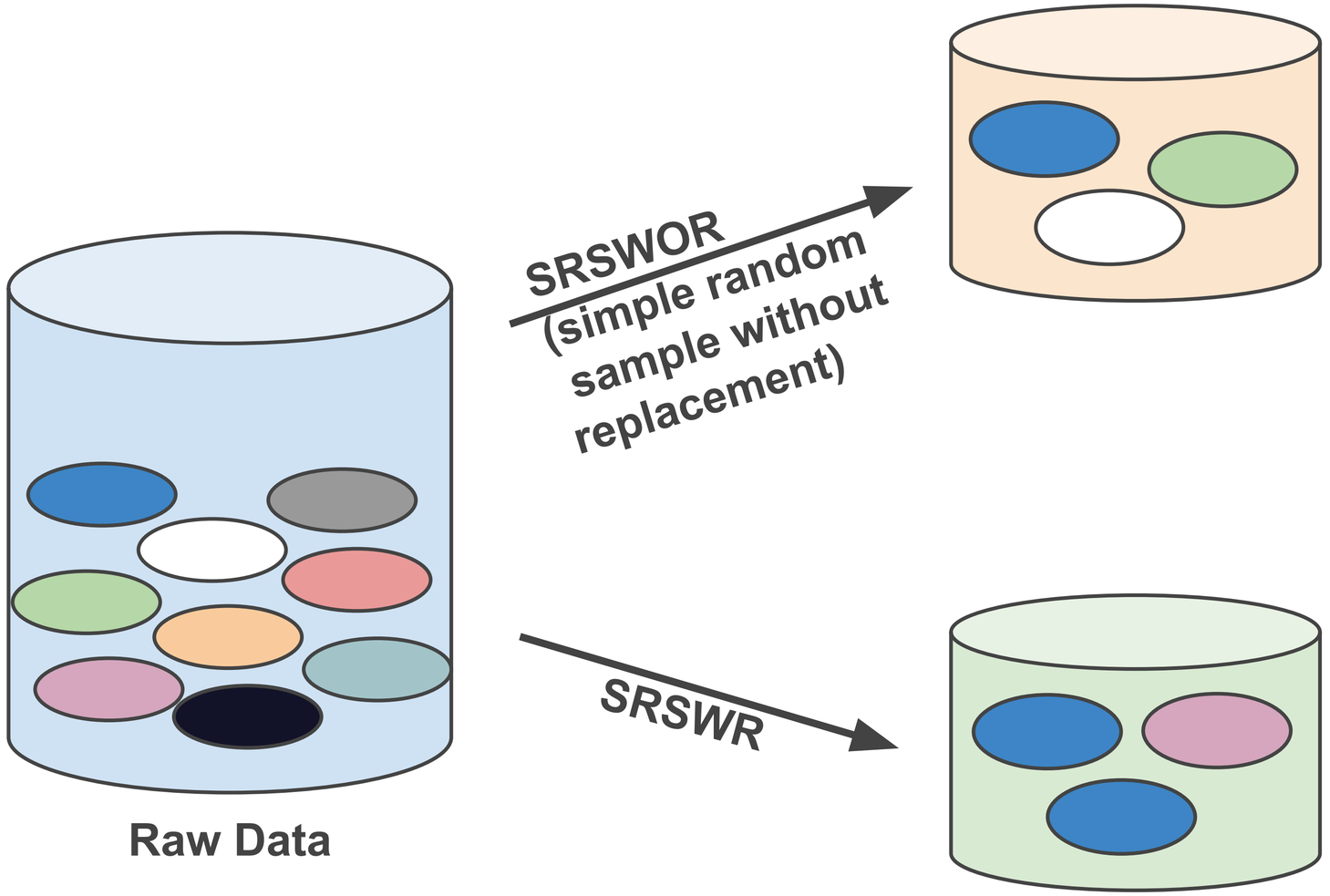
Sampling

- Sampling: obtaining a small sample s to represent the whole data set N
- Allow a mining algorithm to run in complexity that is potentially sub-linear to the size of the data
- Key principle: Choose a representative subset of the data
  - Simple random sampling may have very poor performance in the presence of skew
  - Develop adaptive sampling methods, e.g., stratified sampling

Types of Sampling

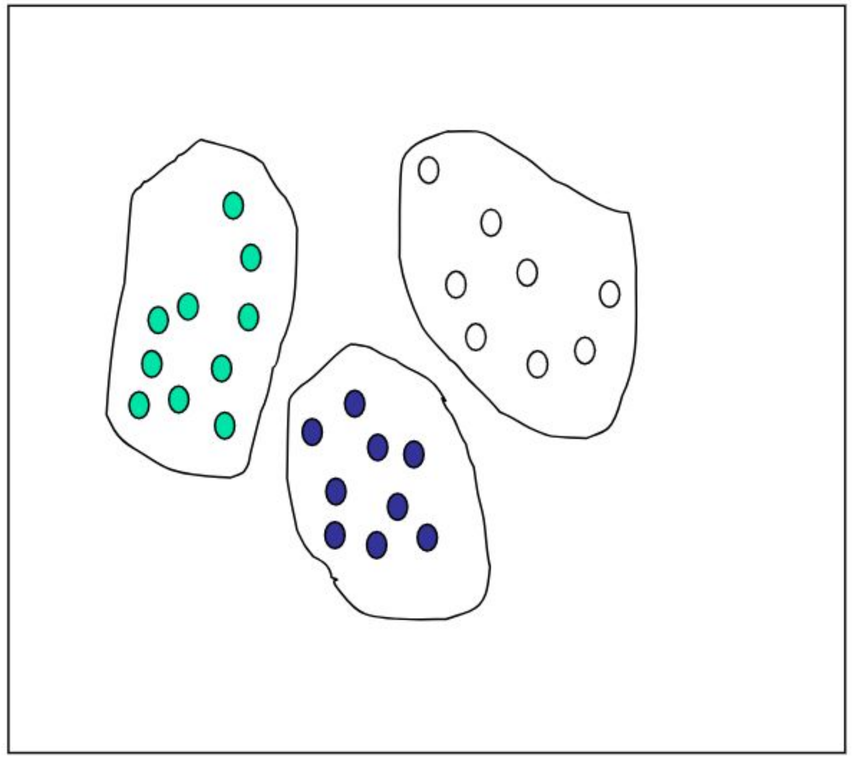
- Simple random sampling
  - There is an equal probability of selecting any particular item
- Sampling without replacement (SRSWOR)
  - Once an object is selected, it is removed from the population
- Sampling with replacement (SRSWR)
  - A selected object is not removed from the population
- Stratified sampling:
  - Partition the data set, and draw samples from each partition (proportionally, i.e., approximately the same percentage of the data)
  - Used in conjunction with skewed data

Sampling: With or Without Replacement

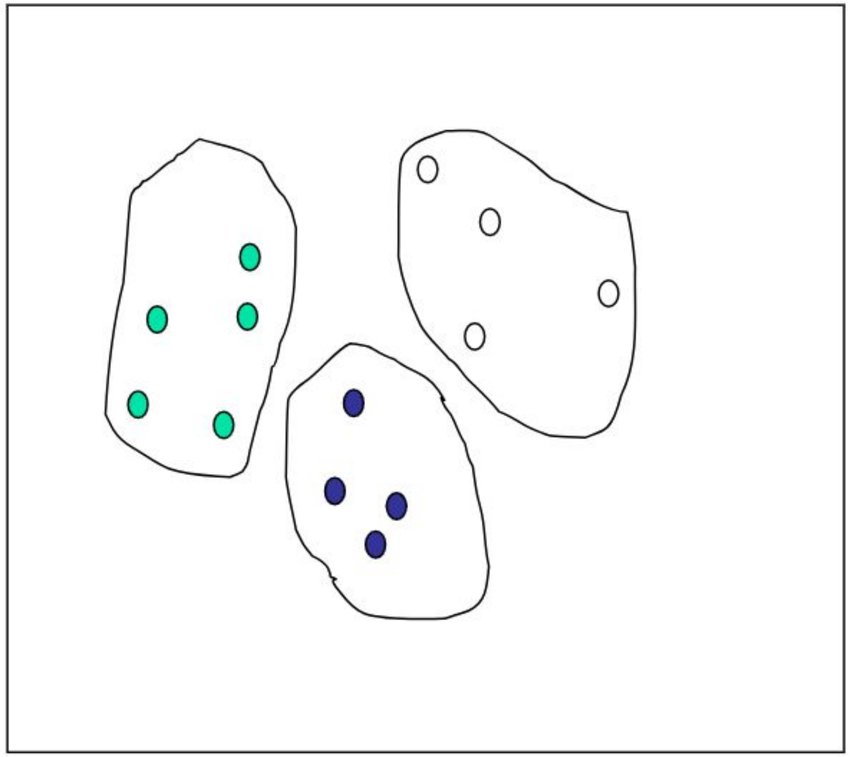


Sampling: Cluster or Stratified Sampling

Raw Data



Cluster/Stratified Sample



Summary

- Data compression
  - Discrete wavelet transform (DWT)
- Sampling
  - Sampling without replacement
  - Sampling with replacement
  - Cluster or Stratified Sampling